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Inference for the Generalized Exponential Distributions with Covariate and Right-Censored Data

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Abstract

The purpose of this paper is to extend the generalized exponential model (GEM) to include covariates in the presence of right-censored data. We obtained the maximum likelihood estimator (MLE) for the parameters of this model. Following that a thorough simulation study was carried out to evaluate the performance of the estimator based on the values of bias, standard error (SE) and root mean square error (RMSE). The results indicated that the SE and RMSE decrease with the increase in sample size and decrease in censoring proportion. Finally, we illustrate the performance of the Wald confidence interval estimation technique for the GE model with right-censored data and covariate by a coverage probability study at several censoring proportions and different sample sizes.

Keywords: generalized exponential distribution; maximum likelihood estimator; fixed covariate; right-censoring; converge probabilit.

1 Introduction

In this paper, our main objective is to study the inferential procedure for the GEM with right-censored and a fixed covariate. In Section 2, we present the materials and methods to obtain the MLE for the GE distribution with right-censored data. Following that, we present a simulation study and discussion in Section 3. Conclusion are finally given in Section 4.

Gupta and Kundu [3] introduced the three-parameter GE distribution and studied the theoretical properties of this family. Their work has shown the GE distribution fits better than the three-parameter Weibull and Gamma distributions in some cases. The GE distribution has the following cumulative distribution function(CDF):

$$F(t;\alpha,\lambda,\mu) = (1-e^{-(t-\mu)/\lambda})^{\alpha}; \quad t>\mu,\ \alpha>0,\ \lambda>0,$$

and probability density function (pdf),

$$f(t;\alpha,\lambda,\mu) = \frac{\alpha}{\lambda}(1-e^{-(t-\mu)/\lambda})^{\alpha-1}e^{-(t-\mu)/\lambda}; \quad t>\mu,\ \alpha>0,\ \lambda>0. \tag{1}$$

Here, α is a shape parameter; λ is a scale parameter, and μ is a location parameter. The Inferential procedure for the GE distribution with censored data developed by Sarhan [12].Works involving the inferences for the GE distribution were done and discussed by authors such as Raqab and Ahsanullah [11], Jaheen [5], Raqab and Madi [10], and Gupta and Kundu [4].

2 Materials and Methods

2.1 The Generalized Exponential Model with Right Censored Data

2.1.1 Maximum Likelihood Estimation Techniques

The MLE can be employed to obtain the parameters estimates for the GEM with right-censored data. First, we consider the three-parameter GE model, and for the sake of simplicity, we reparametrize $\beta = 1/\lambda$.

Then the likelihood of construction for right-censored data for the complete sample with $i = 1, ..., n$ is given by:

$$l = \prod_{i=1}^{n}\left\{f_i(t_i)^{\delta_i}\left[1-F(t_i)\right]^{1-\delta_i}\right\},$$

$$l = \prod_{i=1}^{n}\left\{f_i(t_i)^{\delta_i}\left[S(t_i)\right]^{1-\delta_i}\right\}.$$

The log−likelihood function for the GEM with right-censored data can be obtained by substituting the probability distribution and survivorship function of the model into the log−likelihood function:

$$L = \sum_{i=1}^{n} \left\{ \delta_i \ln \left[\alpha\beta(1 - e^{-\beta(t_i-\mu)})^{\alpha-1} e^{-\beta(t_i-\mu)} \right] + (1-\delta_i) \ln \left[1 - (1 - e^{-\beta(t_i-\mu)})^{\alpha} \right] \right\}, \tag{2}$$

$$\begin{aligned} L &= \sum_{i=1}^{n} \delta_i \left[\ln(\alpha\beta) + (\alpha-1)\ln(1 - e^{-\beta(t_i-\mu)}) - \beta(t_i-\mu) \right. \\ &+ \left. (1-\delta_i) \ln \left[1 - (1 - e^{-\beta(t_i-\mu)})^{\alpha} \right] \right], \end{aligned} \tag{3}$$

where the indicator variable δ_i define as :

$$\delta_i = \begin{cases} 1 & \text{data is uncensored,} \\ 0 & \text{data is right-censored.} \end{cases}$$

2.2 The Generalized Exponential Model with Right Censored Data and Fixed Covariate

In this research, we incorporate a fixed covariate to the GEM to accommodate the effect of covariate on survival time. This is achieved by taking $\beta = e^{b_0+b_1x}$.

$$\begin{aligned} L &= \sum_{i=1}^{n} \delta_i \left[\ln\alpha + \ln e^{(b_0+b_1x)} + (\alpha-1)\ln(1 - e^{-e^{(b_0+b_1x)}(t_i-\mu)}) - e^{(b_0+b_1x)}(t_i-\mu) \right. \\ &+ \left. (1-\delta_i) \ln \left[1 - (1 - e^{-e^{(b_0+b_1x)}(t_i-\mu)})^{\alpha} \right] \right]. \end{aligned} \tag{4}$$

The first derivative of the log likelihood function with respect to the parameters, α, μ, b_0, b_1, are:

$$\frac{\partial L}{\partial \alpha} = \sum_{i=1}^{n} \delta_i \left[\frac{1}{\alpha} + \ln(1 - e^{-e^{(b_0+b_1x)}(t_i-\mu)}) \right] + \sum_{i=1}^{n} (\delta_i - 1) \left[\ln(1 - e^{-e^{(b_0+b_1x)}(t_i-\mu)}) \right].$$

$$\begin{aligned} \frac{\partial L}{\partial \mu} &= \sum_{i=1}^{n} \delta_i \left[e^{(b_0+b_1x)} - (\alpha-1) \frac{e^{b_0+b_1x-e^{(b_0+b_1x)}(t_i-\mu)}}{1 - e^{-e^{(b_0+b_1)}(t_i-\mu)}} \right] \\ &+ \sum_{i=1}^{n} \alpha(1-\delta_i) \frac{e^{(b_0+b_1x)-e^{(b_0+b_1x)}(t_i-\mu)}}{1 - e^{-e^{b_0+b_1x}(t_i-\mu)}}. \end{aligned}$$

$$\frac{\partial L}{\partial b_j} = \sum_{i=1}^{n}\delta_i\left[\frac{x^j}{b_0+b_1x} - \sum_{i=1}^{n} e^{b_0+b_1x}x^j(t_i-\mu) + (\alpha-1)\frac{e^{b_0+b_1x-e^{b_0+b_1x}(t_i-\mu)}x(t_i-\mu)}{1-e^{-e^{b_0+b_1x}(t_i-\mu)}}\right]$$
$$- \sum_{i=1}^{n}(1-\delta_i)\alpha\sum_{i=1}^{n}\frac{e^{b_0+b_1x-e^{b_0+b_1x}(t_i-\mu)}x^j(t_i-\mu)}{1-e^{-e^{b_0+b_1x}(t_i-\mu)}},$$

where $j = 0, 1$.

2.3 Wald Confidence Interval

Wald confidence interval is base on the asymptotic normality of MLE. The implementation of the Wald confidence interval is usually with the maximum likelihood Cox and Hinkley [2]. Let θ be the parameter and $\hat{\theta}$ be maximum likelihood estimate of θ, and $SE(\hat{\theta})$ be the standard error of $\hat{\theta}$. Asymptotically, the Wald statistics for θ follows a standard normal distribution;

$$\hat{\theta} \sim N(\theta, V(\theta)).$$

The $100(1-\alpha)\%$ Wald CI for θ can be constructed as Arasan [1]:

$$\hat{\theta} \pm z_{1-\frac{\alpha}{2}}\sqrt{I^{-1}(\hat{\theta}_{jj})},$$

where $z_{1-\frac{\alpha}{2}}$ is the standard normal score of the $\frac{\alpha}{2}$ th percentile.The covariance matrix denoted by $I^{-1}(\theta)$, where $I(\theta)$ is the Fisher information matrix, measured at the true value of the parameter θ.

3 Simulation Study and Discussion

The simulation study was carried out to assess the performance of parameters estimates of the GE distribution with right-censored data and the fixed covariate. The simulation was conducted by using R software at different combination of sample size n random censoring. Similar simulation approach was used in Olaniran and Yahya [9], Jamil et al. [6], and Olaniran and Abdullah [8].

In this section, first we illustrate the simulation results with the true set parameters $\mu = 0.5$, $\alpha = 0.7$, $b_0 = -1.6$ and $b_1 = -0.3$ is presented. The parameter estimates at various sample sizes as well as the associated the bias,standard error (SE), and root mean square (RMSE) are obtained in Tables 1, 2, and 3 respectively. The results showed that the estimate is precise at various sample sizes.

Table 1 displays the bias of the parameters estimators of different sample sizes and censoring proportions. The value of the bias decreases as the size of the sample increases.

Table 1: Summary table for bias of the parameters for various n and cp.

		Censoring proportion					
Estmiates	Sample size (n)	0%	10%	20%	30%	40%	50%
	30	0.045	0.044	0.043	0.04	0.03	0.023
$\hat{\mu}$	40	0.031	0.03	0.03	0.03	0.029	0.026
	50	0.023	0.023	0.023	0.022	0.022	0.018
	60	0.017	0.018	0.017	0.017	0.017	0.014
	80	0.012	0.011	0.011	0.011	0.011	0.011
	100	0.009	0.009	0.009	0.009	0.009	0.009
	30	-0.039	-0.01	0.023	0.059	0.126	0.208
$\hat{\alpha}$	40	-0.027	-0.005	0.025	0.053	0.082	0.103
	50	-0.022	-0.003	0.026	0.046	0.071	0.101
	60	-0.02	-0.002	0.023	0.048	0.068	0.098
	80	-0.009	0.007	0.029	0.05	0.069	0.082
	100	-0.007	0.008	0.035	0.056	0.073	0.074
	30	0.057	-0.022	-0.122	-0.242	-0.409	-0.633
$\hat{b_0}$	40	0.056	-0.034	-0.129	-0.258	-0.418	-0.637
	50	0.043	-0.041	-0.135	-0.274	-0.438	-0.649
	60	0.041	-0.044	-0.143	-0.272	-0.445	-0.653
	80	0.038	-0.043	-0.15	-0.285	-0.461	-0.66
	100	0.036	-0.046	-0.139	-0.287	-0.459	-0.685
	30	0.011	0.02	0.027	0.027	0.029	0.029
$\hat{b_1}$	40	0.01	0.02	0.023	0.029	0.028	0.008
	50	0.016	0.025	0.023	0.029	0.023	0.025
	60	0.02	0.026	0.023	0.028	0.027	0.028
	80	0.012	0.015	0.018	0.018	0.015	0.019
	100	0.01	0.016	0.017	0.018	0.017	0.016

Besides, Table 2 show the percentage of cp increases, the values of the standard error will continue to increase. The more accurate inference is making by increasing the size of the sampling with low censored data proportions. All parameters of this model show good performance by having a relatively low standard error when the sample size increases and cp decreases. Table 3, the RMSE values also get lower as the sample size increases but hight when the cp is increasing.

Table 2: Summary table for the (SE) of the parameters for various n and cp.

		Censoring proportion					
Estmiates	Sample size	0%	10%	20%	30%	40%	50%
	30	0.066	0.072	0.075	0.089	0.208	0.315
$\hat{\mu}$	40	0.047	0.054	0.053	0.05	0.057	0.113
	50	0.033	0.033	0.033	0.033	0.033	0.102
	60	0.025	0.025	0.025	0.025	0.025	0.098
	80	0.015	0.016	0.016	0.016	0.017	0.017
	100	0.012	0.013	0.013	0.014	0.014	0.013
	30	0.149	0.187	0.225	0.283	0.812	2.049
$\hat{\alpha}$	40	0.123	0.185	0.192	0.195	0.213	0.415
	50	0.107	0.129	0.138	0.154	0.169	0.29
	60	0.097	0.11	0.128	0.137	0.153	0.453
	80	0.082	0.101	0.111	0.123	0.131	0.144
	100	0.07	0.086	0.098	0.11	0.12	0.129
	30	0.227	0.253	0.304	0.335	0.408	0.467
$\hat{b_0}$	40	0.194	0.22	0.243	0.288	0.34	0.409
	50	0.181	0.193	0.209	0.262	0.306	0.369
	60	0.155	0.168	0.2	0.242	0.289	0.33
	80	0.134	0.149	0.179	0.222	0.255	0.287
	100	0.121	0.128	0.16	0.205	0.232	0.262
	30	0.249	0.257	0.276	0.307	0.352	0.389
$\hat{b_1}$	40	0.204	0.219	0.236	0.25	0.288	0.319
	50	0.187	0.197	0.212	0.238	0.253	0.295
	60	0.16	0.173	0.184	0.211	0.232	0.257
	80	0.134	0.148	0.161	0.18	0.198	0.226
	100	0.121	0.132	0.141	0.161	0.181	0.198

Table 3: Summary table for the (RMSE) of the parameters for various n and cp.

Estmiates	Sample size	Censoring proportion					
		0%	10%	20%	30%	40%	50%
	30	0.079	0.084	0.086	0.098	0.21	0.316
$\hat{\mu}$	40	0.056	0.062	0.061	0.058	0.064	0.116
	50	0.04	0.04	0.04	0.04	0.04	0.103
	60	0.03	0.03	0.03	0.03	0.031	0.099
	80	0.02	0.02	0.02	0.02	0.02	0.02
	100	0.016	0.016	0.016	0.016	0.016	0.016
	30	0.154	0.188	0.226	0.289	0.821	2.06
$\hat{\alpha}$	40	0.126	0.185	0.194	0.202	0.228	0.427
	50	0.109	0.129	0.141	0.161	0.184	0.307
	60	0.099	0.11	0.13	0.145	0.167	0.464
	80	0.082	0.101	0.115	0.133	0.148	0.166
	100	0.07	0.086	0.104	0.124	0.141	0.149
	30	0.234	0.254	0.327	0.413	0.577	0.787
$\hat{b_0}$	40	0.202	0.223	0.275	0.387	0.539	0.757
	50	0.186	0.198	0.248	0.379	0.534	0.747
	60	0.16	0.174	0.246	0.364	0.53	0.732
	80	0.139	0.155	0.233	0.361	0.527	0.72
	100	0.126	0.136	0.212	0.353	0.514	0.734
	30	0.249	0.258	0.278	0.308	0.353	0.389
$\hat{b_1}$	40	0.204	0.22	0.238	0.251	0.29	0.319
	50	0.188	0.199	0.213	0.24	0.254	0.296
	60	0.161	0.175	0.186	0.213	0.233	0.259
	80	0.134	0.149	0.162	0.181	0.199	0.226
	100	0.121	0.133	0.142	0.162	0.182	0.199

3.1 Coverage Probability Study

The coverage probability is the proportion of time a confidence interval includes the true population parameter value. Here, we study the coverage probability for the Wald confidence interval using a coverage probability study. In this study, 2000 replications of size $n = 30, 40, 50, 60, 80$ and 100 were simulated. The nominal Type I error α was set to $\alpha = 0.05$ and $\alpha = 0.10$. The approach of Manoharan et al. [7] was adopted to calculate the error probabilities from the left (lep) and right (rep) and the corresponding total error probability (tep) which is the sum of lep and rep. Upon computing tep, the Wald confidence is termed anticonservative if $tep > \alpha + 2.58s.e.(\hat{\alpha})$, conservative (C) if $tep < \alpha + 2.58s.e.(\hat{\alpha})$ with $s.e.(\hat{\alpha}) = \sqrt{\alpha(1-\alpha)/N}$. For the asymmetric (AS), the larger error probability will be 1.5 times the smaller one. The Wald confidence interval is optimal if AC, C and AS are close to zero and lep, rep and tep are close $0.025(0.05)$ and $0.05(0.10)$ respectively.

Tables 4 - 5 show the results of estimated error probabilities for all parameter using the Wald over different sample sizes and censoring proportions at 5% and 10% significance levels. The results show that the lep and rep are asymmetrical under most settings i.e, at various sample sizes and censoring proportions, the lep or rep are 1.5 times larger than the other for all parameters. However, the tep values are closer to the nominal 0.05 and 0.1 at all settings for parameter b_0 and b_1.

Table 4: Estimated error probabilities for all parameter using the Wald over different sample sizes and censoring proportions at 5% significance level.

Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
0	30	0.016	0.002	0.018	0.001	0.000	0.001	0.042	0.021	0.063	0.028	0.033	0.060
	40	0.100	0.001	0.100	0.011	0.054	0.064	0.027	0.028	0.055	0.029	0.026	0.054
	50	0.076	0.000	0.076	0.018	0.041	0.059	0.043	0.020	0.062	0.027	0.028	0.055
	60	0.073	0.001	0.074	0.020	0.033	0.053	0.041	0.023	0.064	0.029	0.025	0.054
	80	0.089	0.000	0.089	0.029	0.041	0.070	0.043	0.016	0.058	0.035	0.022	0.057
	100	0.084	0.000	0.084	0.033	0.032	0.065	0.041	0.021	0.061	0.035	0.021	0.056
0.1	30	0.012	0.003	0.015	0.003	0.000	0.003	0.015	0.048	0.063	0.030	0.027	0.056
	40	0.084	0.002	0.086	0.022	0.016	0.038	0.011	0.049	0.060	0.026	0.025	0.051
	50	0.084	0.001	0.085	0.024	0.024	0.048	0.014	0.053	0.067	0.033	0.022	0.054
	60	0.095	0.001	0.095	0.027	0.019	0.046	0.010	0.063	0.073	0.021	0.029	0.050
	80	0.087	0.000	0.087	0.033	0.015	0.048	0.010	0.064	0.073	0.033	0.025	0.058
	100	0.087	0.000	0.087	0.040	0.020	0.060	0.008	0.071	0.078	0.034	0.017	0.051
0.2	30	0.009	0.006	0.015	0.005	0.000	0.005	0.007	0.070	0.077	0.031	0.028	0.059
	40	0.086	0.001	0.087	0.036	0.008	0.043	0.006	0.088	0.094	0.026	0.030	0.055
	50	0.081	0.002	0.083	0.032	0.012	0.044	0.004	0.101	0.105	0.037	0.022	0.059
	60	0.094	0.001	0.095	0.033	0.010	0.042	0.003	0.126	0.013	0.028	0.028	0.055
	80	0.087	0.000	0.087	0.036	0.009	0.045	0.002	0.134	0.136	0.028	0.025	0.053
	100	0.087	0.000	0.087	0.059	0.007	0.066	0.001	0.149	0.149	0.036	0.018	0.054
0.3	30	0.002	0.005	0.006	0.003	0.000	0.003	0.003	0.106	0.109	0.036	0.027	0.062
	40	0.080	0.003	0.083	0.047	0.002	0.048	0.001	0.141	0.142	0.028	0.021	0.049
	50	0.073	0.003	0.075	0.042	0.003	0.044	0.001	0.165	0.166	0.035	0.020	0.054
	60	0.094	0.001	0.095	0.045	0.002	0.047	0.000	0.196	0.196	0.030	0.029	0.059
	80	0.087	0.000	0.087	0.058	0.004	0.061	0.000	0.239	0.239	0.033	0.025	0.057
	100	0.087	0.000	0.087	0.073	0.005	0.077	0.000	0.278	0.278	0.035	0.017	0.052
0.4	30	0.001	0.006	0.006	0.004	0.000	0.004	0.002	0.175	0.176	0.032	0.026	0.058
	40	0.015	0.003	0.017	0.009	0.000	0.009	0.001	0.222	0.223	0.001	0.222	0.223
	50	0.073	0.006	0.078	0.049	0.001	0.050	0.000	0.285	0.285	0.031	0.026	0.057
	60	0.094	0.001	0.094	0.059	0.002	0.060	0.000	0.348	0.348	0.032	0.027	0.059
	80	0.086	0.001	0.087	0.069	0.001	0.070	0.000	0.410	0.410	0.037	0.019	0.055
	100	0.087	0.000	0.087	0.085	0.000	0.085	0.000	0.489	0.489	0.038	0.016	0.054
0.5	30	0.001	0.008	0.009	0.007	0.000	0.007	0.000	0.225	0.225	0.030	0.023	0.053
	40	0.050	0.005	0.055	0.049	0.000	0.049	0.000	0.335	0.335	0.028	0.029	0.057
	50	0.001	0.002	0.002	0.004	0.000	0.004	0.000	0.380	0.380	0.032	0.025	0.057
	60	0.045	0.003	0.048	0.065	0.001	0.066	0.000	0.506	0.506	0.034	0.021	0.054
	80	0.085	0.002	0.087	0.079	0.001	0.080	0.000	0.644	0.644	0.035	0.021	0.056
	100	0.086	0.001	0.087	0.081	0.001	0.081	0.000	0.732	0.732	0.035	0.023	0.058

Table 5: Estimated error probabilities for all parameter using the Wald method over different sample sizes and censoring proportions at 10% significance level.

Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
0	30	0.028	0.003	0.030	0.001	0.000	0.001	0.081	0.034	0.114	0.051	0.048	0.099
	40	0.130	0.001	0.131	0.023	0.098	0.121	0.058	0.038	0.096	0.050	0.044	0.094
	50	0.101	0.000	0.101	0.033	0.074	0.107	0.070	0.034	0.104	0.047	0.046	0.093
	60	0.099	0.001	0.100	0.033	0.060	0.093	0.069	0.034	0.103	0.058	0.044	0.102
	80	0.127	0.000	0.127	0.046	0.067	0.112	0.078	0.030	0.108	0.059	0.040	0.099
	100	0.105	0.000	0.105	0.055	0.059	0.114	0.080	0.031	0.111	0.063	0.032	0.094
0.1	30	0.018	0.003	0.021	0.003	0.000	0.003	0.036	0.066	0.102	0.056	0.048	0.103
	40	0.115	0.002	0.117	0.033	0.044	0.076	0.026	0.765	0.102	0.051	0.046	0.097
	50	0.111	0.001	0.112	0.043	0.051	0.094	0.028	0.079	0.107	0.062	0.035	0.097
	60	0.128	0.001	0.128	0.042	0.053	0.095	0.018	0.093	0.110	0.052	0.047	0.099
	80	0.119	0.000	0.119	0.060	0.038	0.098	0.018	0.088	0.106	0.059	0.044	0.103
	100	0.111	0.000	0.111	0.071	0.043	0.114	0.015	0.102	0.116	0.066	0.034	0.100
0.2	30	0.014	0.007	0.021	0.008	0.000	0.008	0.014	0.108	0.122	0.056	0.044	0.100
	40	0.116	0.001	0.116	0.048	0.021	0.068	0.011	0.130	0.141	0.049	0.048	0.097
	50	0.103	0.002	0.105	0.051	0.025	0.076	0.008	0.139	0.147	0.063	0.040	0.102
	60	0.127	0.001	0.128	0.060	0.029	0.089	0.003	0.171	0.174	0.049	0.049	0.098
	80	0.120	0.000	0.120	0.086	0.023	0.109	0.002	0.187	0.189	0.058	0.044	0.101
	100	0.111	0.000	0.111	0.105	0.017	0.122	0.001	0.211	0.212	0.061	0.035	0.095
0.3	30	0.004	0.005	0.009	0.004	0.000	0.004	0.006	0.162	0.168	0.060	0.041	0.100
	40	0.110	0.003	0.113	0.064	0.012	0.076	0.003	0.203	0.206	0.055	0.039	0.094
	50	0.096	0.004	0.100	0.062	0.012	0.074	0.002	0.233	0.235	0.067	0.037	0.104
	60	0.127	0.001	0.128	0.082	0.014	0.096	0.000	0.279	0.279	0.058	0.042	0.100
	80	0.119	0.001	0.119	0.108	0.012	0.120	0.000	0.323	0.323	0.054	0.040	0.094
	100	0.110	0.000	0.110	0.134	0.011	0.145	0.000	0.375	0.375	0.066	0.039	0.105
0.4	30	0.001	0.006	0.007	0.004	0.000	0.004	0.003	0.251	0.251	0.254	0.057	0.040
	40	0.022	0.003	0.024	0.014	0.000	0.014	0.001	0.321	0.322	0.055	0.045	0.099
	50	0.096	0.006	0.102	0.073	0.006	0.078	0.001	0.382	0.382	0.055	0.043	0.098
	60	0.127	0.001	0.128	0.092	0.008	1.000	0.000	0.458	0.458	0.057	0.044	1.000
	80	0.115	0.002	0.117	0.132	0.007	0.139	0.000	0.522	0.522	0.064	0.038	0.102
	100	0.110	0.000	0.110	0.155	0.006	0.161	0.000	0.608	0.608	0.064	0.032	0.096
0.5	30	0.001	0.009	0.010	0.008	0.000	0.008	0.001	0.329	0.329	0.052	0.041	0.093
	40	0.073	0.005	0.078	0.065	0.000	0.065	0.000	0.466	0.466	0.048	0.051	0.099
	50	0.001	0.002	0.002	0.006	0.000	0.006	0.000	0.499	0.499	0.053	0.045	0.098
	60	0.070	0.003	0.073	0.101	0.003	0.104	0.000	0.635	0.635	0.058	0.043	0.101
	80	0.114	0.002	0.116	0.136	0.006	0.142	0.000	0.763	0.763	0.062	0.040	0.101
	100	0.111	0.001	0.112	0.140	0.003	0.143	0.000	0.827	0.827	0.064	0.038	0.102

Tables 6 - 7 show the results for total number of AC, C, and AS intervals for all parameters using Wald intervals over varying sample sizes and censored proportions and Type I error 0.05 and 0.10. The results show that the Wald intervals are anti-conservative for parameters m and a at most settings but asymmetric for b_0 and b_1.

Table 6: Total number of AC, C, and AS intervals for all parameters using Wald intervals over varying sample sizes and censored proportions and Type I error 0.05.

α		0.05											
Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b}_0$			$\hat{b}_1$		
cp	n	AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
0	30	3	1	4	0	2	4	2	0	2	1	0	0
	40	4	0	4	2	0	4	1	0	2	0	0	0
	50	4	0	4	1	0	4	0	0	2	0	0	1
	60	4	0	4	1	0	4	1	0	3	0	0	0
	80	4	0	4	4	0	1	3	0	4	0	0	2
	100	4	0	4	4	0	0	3	0	4	0	0	2
0.1	30	1	2	4	0	4	4	0	0	4	0	0	1
	40	3	0	4	0	2	1	1	0	4	0	0	0
	50	4	0	4	0	0	1	4	0	4	0	0	1
	60	4	0	4	0	0	2	3	0	4	0	0	1
	80	4	0	4	0	0	4	4	0	4	0	0	1
	100	4	0	4	1	0	4	4	0	4	0	0	3
0.2	30	0	1	3	0	4	4	4	0	4	0	0	1
	40	4	0	4	0	2	4	4	0	4	0	0	1
	50	4	0	4	0	1	4	4	0	4	0	0	2
	60	4	0	4	0	0	4	4	0	4	0	0	1
	80	4	0	4	0	0	4	4	0	4	0	0	2
	100	4	0	4	2	0	4	4	0	4	0	0	3
0.3	30	0	3	4	0	4	4	4	0	4	0	0	0
	40	2	0	4	0	2	4	4	0	4	0	0	0
	50	3	0	4	0	1	4	4	0	4	0	0	1
	60	4	0	4	0	0	4	4	0	4	0	0	1
	80	4	0	4	1	0	4	4	0	4	0	0	2
	100	4	0	4	4	0	4	4	0	4	0	0	3
0.4	30	0	4	4	0	4	4	4	0	4	0	0	2
	40	1	2	4	0	2	4	4	0	4	0	0	1
	50	4	0	4	0	0	4	4	0	4	0	0	2
	60	4	0	4	0	0	4	4	0	4	0	0	1
	80	4	0	4	4	0	4	4	0	4	0	0	4
	100	4	0	4	4	0	4	4	0	4	0	0	4
0.5	30	0	4	3	0	4	4	4	0	4	1	0	1
	40	0	3	4	0	3	4	4	0	4	0	0	0
	50	2	1	4	3	1	4	4	0	4	0	0	2
	60	1	1	4	1	0	4	4	0	4	0	0	4
	80	4	0	4	4	0	4	4	0	4	0	0	2
	100	4	0	4	4	0	4	4	0	4	0	0	2

Table 7: Total number of AC, C, and AS intervals for all parameters using Wald intervals over varying sample sizes and censored proportions and Type I error 0.1.

α		0.10											
Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
0	30	1	1	4	0	2	4	0	0	3	0	0	0
	40	4	0	4	1	0	4	0	0	4	0	0	0
	50	2	0	4	1	0	4	0	0	4	0	0	0
	60	2	0	4	1	0	4	0	0	4	0	0	0
	80	2	0	4	1	0	1	1	0	4	0	0	1
	100	1	0	4	0	0	0	0	0	4	0	0	3
0.1	30	0	2	4	0	4	4	0	0	4	0	0	1
	40	1	1	4	0	3	1	0	0	4	0	0	0
	50	1	0	4	0	0	0	0	0	4	0	0	2
	60	2	0	4	0	0	0	0	0	4	0	0	1
	80	3	0	4	0	0	3	1	0	4	0	0	1
	100	2	0	4	0	0	4	0	0	4	0	0	2
0.2	30	0	2	4	0	4	4	4	0	4	0	0	0
	40	1	0	4	0	4	4	4	0	4	0	0	1
	50	0	0	4	0	2	4	4	0	4	0	0	1
	60	2	0	4	0	1	4	4	0	4	0	0	1
	80	2	0	4	0	0	4	4	0	4	0	0	2
	100	2	0	4	3	0	4	4	0	4	0	0	3
0.3	30	0	4	2	0	4	4	4	0	4	0	1	1
	40	1	1	4	0	4	4	4	0	4	0	0	0
	50	1	1	4	0	3	4	4	0	4	0	0	1
	60	2	0	4	0	1	4	4	0	4	0	0	2
	80	3	0	4	2	0	4	4	0	4	0	0	1
	100	2	0	4	4	0	4	4	0	4	0	0	3
0.4	30	0	0	4	0	0	4	4	0	4	0	0	1
	40	1	3	4	0	4	4	4	0	4	0	0	2
	50	0	0	4	0	3	4	4	0	4	0	0	2
	60	2	0	4	0	0	4	4	0	4	0	0	2
	80	1	0	4	4	0	4	4	0	4	0	0	4
	100	1	0	4	4	0	4	4	0	4	0	0	4
0.5	30	0	4	3	0	4	4	4	0	4	0	0	1
	40	0	4	3	0	4	4	4	0	4	0	0	0
	50	0	2	4	0	1	4	4	0	4	0	0	2
	60	0	3	4	0	1	4	4	0	4	0	0	2
	80	1	0	4	4	0	4	4	0	4	0	0	2
	100	0	0	4	4	0	4	4	0	4	0	0	3

3.2 Real Data

When comparing the GE model with classical exponential distribution using lung cancer survival dataset was reported in Table 8 and Table 9 for the cases of without and with a covariate. The diagonal elements of the matrix correspond to the estimated variance. Notice for the variance of $\hat{\mu}$ is large and this confirms that the estimate of μ is not efficient when $\alpha < 2$. The log-likelihood estimate (LL) in Tables 8 & 9 show that the GE model provides a better fit for the Lung cancer data than standard exponential distribution.

Table 8: Comparison result with exponential distribution without covariate.

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	LL
Exponential	-	0.002	-	-848.023
GE	1.510	0.003	0.457	-842.255
SE	0.2355	0.000	6.981	-

Table 9: Comparison result with exponential distribution with covariate.

	$\hat{\alpha}$	$\hat{\mu}$	$\hat{b}_0$	$\hat{b}_1$	LL
Exponential	-	-	7.197	-0.018	-846.530
GE	1.416	0.911	6.684	-0.014	-840.680
SE	0.24021	7.3731	0.5775	0.01000	-

4 Conclusion

In this paper, we have considered the (GEM) with fixed covariate and right-censored data. Results a simulation study shows that the bias, standard error, and root mean square error are higher censoring proportion, and smaller sample sizes. That indicates the estimates perform better when the sample size is larger, and censoring proportion is lower. On the other hand, the total error probability for parameters b_0 and b_1 with the Wald method is closer to the nominal level of $\alpha = 0.05$ and $\alpha = 0.1$ with symmetrical left error probability and right error probability. When the coverage probability for the Wald confidence interval appears higher error probability for low sample size, and lower error probability for a high sample size. Also, for censoring proportion, lower error probability for low censoring proportion, and higher error probability for high censoring proportion.

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Conflicts of Interest The authors declare no conflict of interest.

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